Multiple-Task Radar Resources Allocation

Joseph Z. Ben-Asher

Technion, ITT, Haifa Israel 32000

and

Dror Cohen

Wales, Ltd., Ramat-Gan, Israel 52522

Abstract

The problem of optimal allocation of radar resources to multiple tasks is addressed. For Ballistic Missile Defense (BMD) system a single performance measure is proposed. The tasks of tracking targets, searching for new targets and targets discrimination in clusters are considered. Optimal strategies are obtained by direct optimization via the Cross–Entropy method. The structure of the solutions is investigated by extensive numerical solutions and the main features of the optimal strategies are characterized.

1. Introduction

A Ballistic Missile Defense (BMD) system uses its phased array radar(s) for various missions such as search, tracking, discrimination (i.e. target identification in cluster), etc.

Missions "compete" over the same finite stockpile of sensor resources and have to be performed within certain time intervals. Mission performance level depends on the amount of sensor resources allocated to it, and therefore can be optimized by specific allocation.

An interim problem of the general resource optimization problem is radar tracking beam allocation. Classically, the objective of the sensor allocation process in tracking has been to minimize the uncertainty in the tracking estimation error of all relevant targets, using a given amount of radar resources. This problem has been addressed by the present writers in Ref [1] where open-loop optimal strategies were obtained by direct optimization. The structure of the solutions was investigated by extensive numerical solutions and the main features of the optimal strategies were characterized.

The goal of the current study is to address the more general resource optimization problem of radar beam allocation for search and track and discrimination missions. To this end various methods have been employed in the past. Wintenby[2] proposed a hierarchic system where the decisions made on resources are separated into two levels. All tasks of radar, e.g. tracking a target, searching for new targets, etc., are assigned to different measurement processes that decide locally which measurement should be done in order to maintain a task. A supervisory resource controller distributes resources on a time averaged basis among the measurement processes, to maximize the predicted performance. Kuo et al [3] characterized the various radar tasks and their priorities (e.g. high priority search, low priority search, normal track, high-precision track, etc.) A real time radar dwell scheduling for the various tasks is proposed to improve the performance based on a known rate-based scheduling. Van-Kuek and Blackman[4] characterize the various missions by their main parameters such as revisit times, dwell times, detection thresholds, etc. These discrete parameters are then used for the optimization. The total requested energy allocated for each track is minimized, and the search task get the residual energy.

The main contribution of the present paper is the proposed methodology of combining the different tasks into a single performance index. This novel idea enables solving the optimization problem as a whole. One can use any direct optimization method to optimally allocate the resources in a quasi-continuous way. The structure of the solutions will be investigated and the main features of the optimal strategies will be characterized.

2. Modeling

The dynamic modeling of the ballistic target and the radar measurements are given in Appendix 1. For simplicity we consider 2 dimensional cases but the methodology can be extended to the more general 3 dimensional case.

When radar measurements are taken under tracking mode over a small time increment ΔT, we get the following approximation for the improvement in the current covariance matrix Φ [5] :

$$
\begin{aligned}
&M = A\Phi A^T + LQL^T \\
&\Phi = M - MC^T\left[CMC^T + R\right]^{-1}CM \\
&Q = \begin{bmatrix} \sigma_{w_1}^2 & 0 \\ 0 & \sigma_{w_2}^2 \end{bmatrix} \\
&R = \frac{1}{u}\begin{bmatrix} \sigma_\theta^2 & 0 \\ 0 & \sigma_r^2 \end{bmatrix}; \quad 0 \le u \le 1
\end{aligned}
\tag{1}
$$

A, C and L are given in Appendix A. σ_θ^2 and σ_r^2 are the measurement noise variances, whereas $\sigma_{w_1}^2$ and $\sigma_{w_2}^2$ are process noise variances. The "control" variable u is the percentages of radar resources (i.e. dwells of the radar tracking beam) allocated to this track mission. Notice that if we track the target over ΔT with full resources, i.e. maximal number of beam dwells, then u=1 and we obtain the measurements covariance matrix as follows

$$R = \begin{bmatrix} \sigma_\theta^2 & 0 \\ 0 & \sigma_r^2 \end{bmatrix} \quad \text{(A4)}$$

However u<1 implies an inversely proportional increase in R. For example, when u=0.5 the variances are doubled.

Remark: Essentially we do here is to approximate the tracking problem by concentrating all measurements (taken over the small time interval ΔT) at the end of ΔT and by taking their mean value. This is done in order to reduce the number of unknowns in the optimization process. Thus we can use u as our control-like variable rather than dealing with each single beam dwells (there may be hundreds of them over the time of interest).

In the prediction (when no measurements are taken) process we use

$$\begin{aligned} M &= A\Phi A^T + LQL \\ \Phi &= M \end{aligned} \quad (2)$$

Finally the predicted position uncertainty is

$$RMS = \sqrt{\Phi_{11}^2 + \Phi_{22}^2} \quad (3)$$

3. Problem Definition

The first step in tackling the multi-task resource optimization problem is radar beam allocation for *search* and *track* missions. Later on we will also combine the *discrimination* mission. We assume that the probabilities of new targets to appear are known (or, at least, can be estimated). In this case we can define the probability of success regarding the missions at hand, as follows:

$$P_{success} = \left(P_{track_1} \cdot P_{track_2} \cdot \ldots P_{track_n}\right) \cdot \left(P_{0new} + P_{1new} \cdot P_d + P_{2new} \cdot P_d^2 + \ldots\right) \quad (4)$$

Where:

P_{inew} for i=1,2,… is the probability of appearance of precisely i new targets.

P_{0new} is the probability that no new target will appear and only old targets (i.e. targets being tracked at the beginning of the time of interest) are to be handled (Notice that $P_{0\,new} + P_{1\,new} + P_{2\,new} + ... = 1$).

Pd is the detection probability of a new target (provided it has materialized).

P_{tracki} is the success probability in achieving the required prediction accuracy for the i[th] target; *n* is the number of tracked targets.

Fig. 1 presents a typical curve connecting resources with probability of detection. The resources are expressed in terms of percentages of radar occupation time over a finite period of time which by itself depends on the size of the search zone.

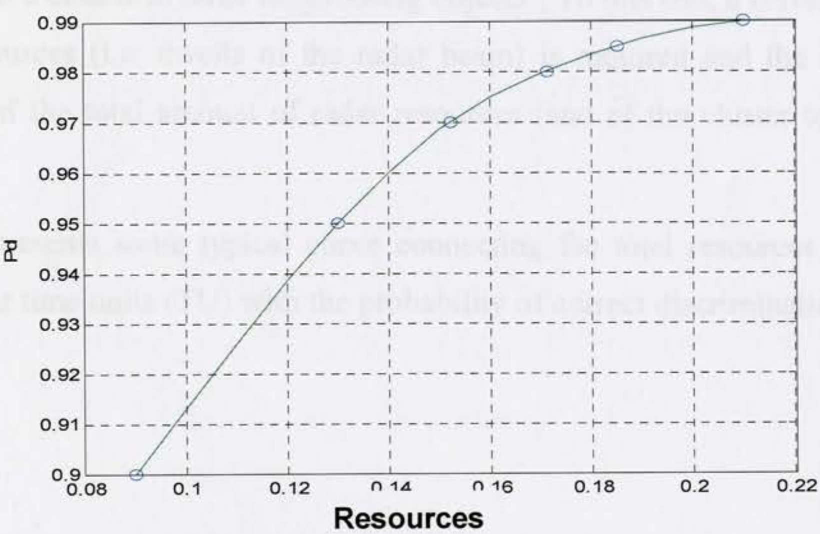

Fig. 1 – P_d vs. Resources

To evaluate the prediction accuracy we will assume the Rayleigh distribution for approximating the predicted position uncertainty (in essence this implies that we have equal values for the variances in the two axes of our 2D tracking problem[5]).

Let R_c be the critical value for the required predicted accuracy (i.e. the tracking mission is successful if its prediction is smaller than or equals to R_c), we then obtain

$$P_{track} = 1 - e^{-R_C^2/2S^2} \quad ; \quad 2S^2 = RMS^2 \qquad (5)$$

RMS is given in (3) and is a function of the allocated resources.

Notice that assuming all other tasks of the defense system are performed with probability 1, the value in (4) represents the probability of zero leakage which is a common measure of performance for BMD systems.

An important radar task in case of BMD is to discriminate between the actual warhead in a cluster to other neighboring objects[6]. To this end, a certain amount of radar resources (i.e. dwells of the radar beam) is required and the success is a function of the total amount of radar resources (and of the cluster to radar slant range).

Fig. 2 represents some typical curve connecting the total resources in terms of some radar time units (TU) with the probability of correct discrimination.

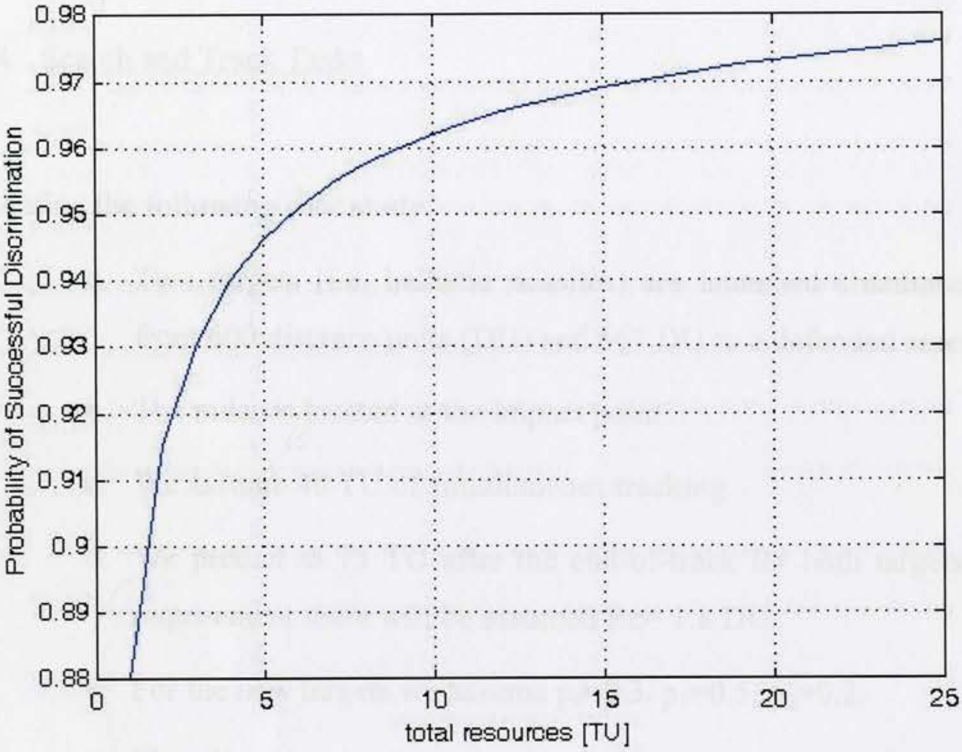

Fig. 2 – $P_{discrimination}$ vs. Total Resources

When discrimination tasks are also considered, we get the next performance measure

$$P_{success} = \left(P_{track_1} \cdot P_{track_2} \cdot \ldots P_{track_n}\right) \cdot \left(P_{0new} + P_{1new} \cdot P_d + P_{2new} \cdot P_d^2 + \ldots\right) \cdot \left(P_{discrimination_1} \cdot P_{discrimination_2} \cdot \ldots P_{discrimination_l}\right) \quad (6)$$

l is the number of targets under discrimination (we assume that they are not contained in the set of tracked targets).

Notice again that assuming all other tasks of the defense system are performed with probability 1, the value in (6) represents the probability of zero leakage.

4. Search and Track Tasks

We define the following case study:

a. Two targets (i.e. ballistic missiles) are launched simultaneously from 600 distance units (DU) and 667 DU to a defended asset
b. The radar is located at the impact point.
c. We assume 40 TU of simultaneous tracking
d. We predict to 73 TU after the end-of-track for both targets; The requirement there will be assumed Rc= 1.8 DU.
e. For the new targets we assume p_0=0.3, p_1=0.5, p_2=0.2.
f. The allocation is determined once at the beginning of the future time segment.
g. We split this time segment to 4 time sub-segments for the search policy (fixed in each sub-segment). Pd in (4) is the average detection probability.
h. Tracking allocation can be changed every 1 TU whereas search allocation (a longer process) can be changed every 10 TU.
i. We shall test for the case where full resources are available and the case when only 50% are available.

Note that at each 1 TU time slot we track a single target. The optimization problem has been solved using Cross-Entropy algorithm[7] which is well suited to this kind of problems.

Fig. 3 and Fig. 4 present the resulting allocation of resources between the two targets, when 100% of the radar resources are available for treating the missions defined above:

Fig. 3 presents the resources allocated for search. One may see that the algorithm decided to allocate ~20% resources for search, and that slightly more resources are allocated to it in the middle periods.

Fig. 4 presents the resources allocated to track of the first target (upper chart) and second target (lower chart). The following observations may be made:

a. The reason for allocating fewer resources to search in the start and end of the tracking period is to allow some more resources for tracking in these sub-segments. This has been analyzed in Ref.1 and was referred to as "Max-Min-Max" pattern

b. The second target, which is more difficult to track (larger distance), receives both more resources, and more of the "sweet spots" (the allocations near the beginning and end of the tracking period). Both targets are converged to an accuracy of about 0.87 DU (not shown in the figures).

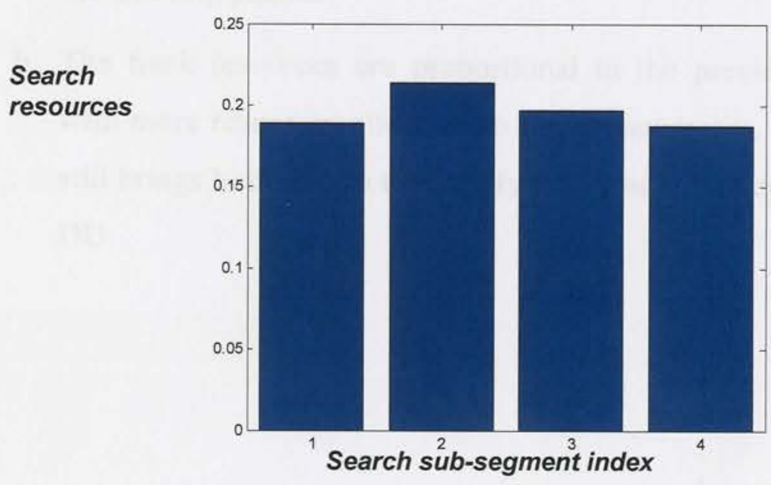

Fig 3 – Search Resources when 100% Resources Available for S&T

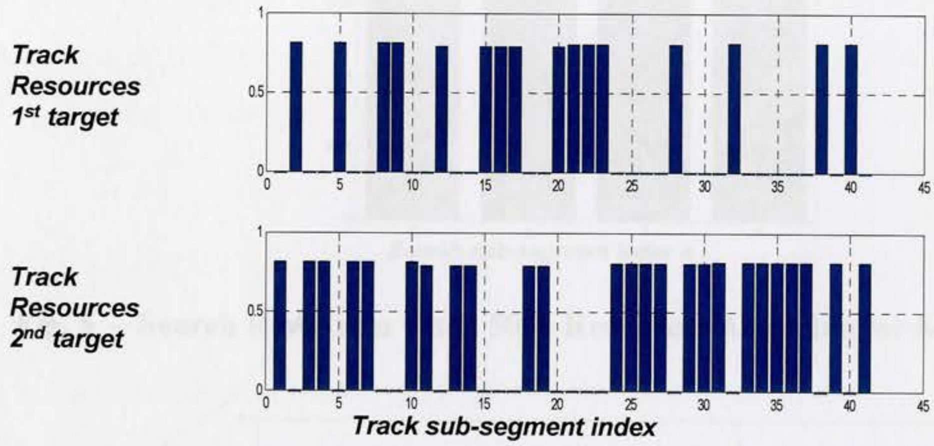

Fig.4 – Track Resources when 100% Resources Available for S&T

We look next at a tougher situation, in which we only allow 50% of radar resources to be allocated for our missions (the rest are supposed to be required by other tasks). Fig. 5 and Fig. 6 present the results:

a. The "Max-Min-Max" tracking structure is more pronounced, with fewer resources allocated to search at the start and end portions of the tracking period.

b. The track resources are proportional to the previous allocation, with more resources allocated to the distant target. The algorithm still brings both targets to roughly the same accuracy of about 1.1 DU.

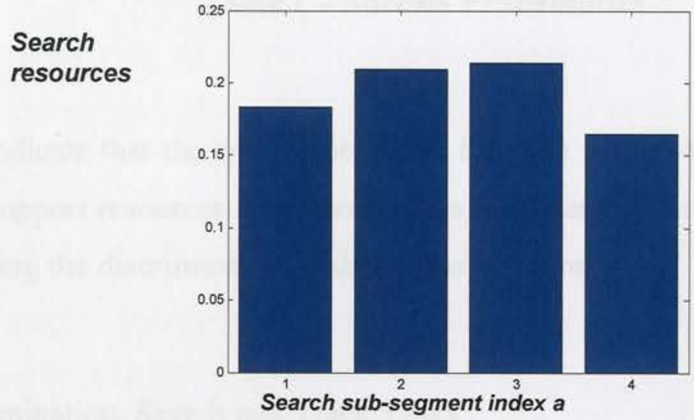

Fig. 5 – Search Resources when 50% Resources Available for S&T

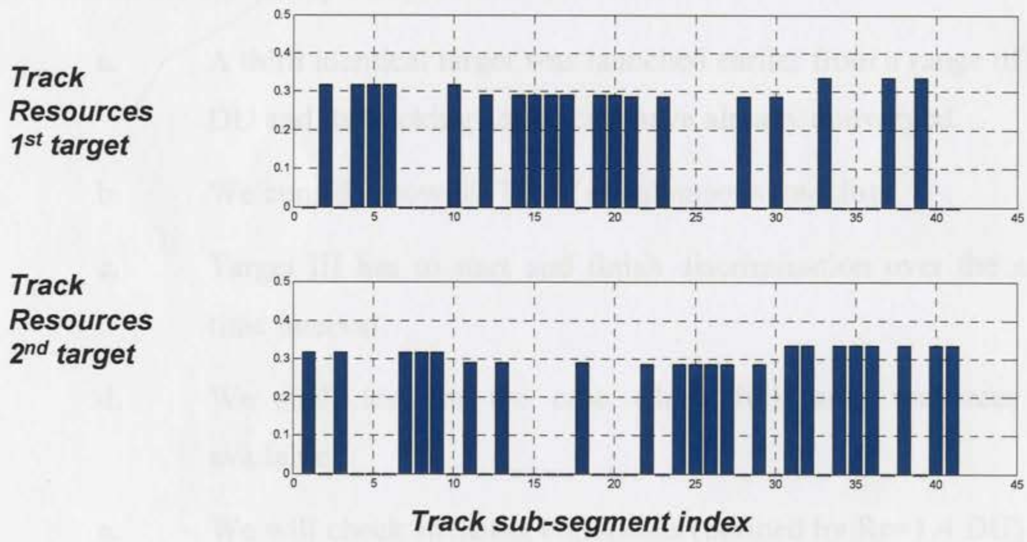

Fig. 6 – Track Resources when 50% Resources Available for S&T

The following table summarizes the results for the case study

Resources	Pd	P_{track1}	P_{track2}	$P_{success}$
100%	0.9880	0.9994	0.9988	0.9875
50%	0.9871	0.9922	0.9862	0.9671

Table 1 – Success Probabilities

The results indicate that the type of objective function we have defined has the potential to support resources allocation across radar tasks. Our next step in the study is to bring the discrimination task into the equation.

5. Discrimination, Search and Track Tasks

We extend the case study analyzed above as follows:

a. A third identical target was launched earlier from a range of 667 DU and its tracking accuracies have already converged.

b. We consider now 60 TU of simultaneous tracking.

c. Target III has to start and finish discrimination over the same time interval.

d. We shall test for the case where full radar resources are available.

e. We will check stressing conditions (defined by Rc=1.4 DU) and benign conditions (Rc=2.1 DU).

f. We split this time to 4 quarters for the search policy.

g. Thus tracking / discrimination allocation can be changed every TU whereas search allocation (a longer process) can be changed every 15 TU.

Fig. 7-9 present the results under stressing conditions. Fig. 7 presents the tracking resources allocation for the two targets under tracking, Fig. 8 shows the allocation for the target under discrimination and Fig. 9 presents the resources allocation for search.

The following observations may be made regarding these results:

a. Whenever resources are scarce, tracking reverts to an almost pure "Max-Min-Max"
b. Discrimination takes the "Min" period for its resources.
c. Search is minimized in the ending sequence, and mostly occupies (together with discrimination) the "Min" tracking zone.

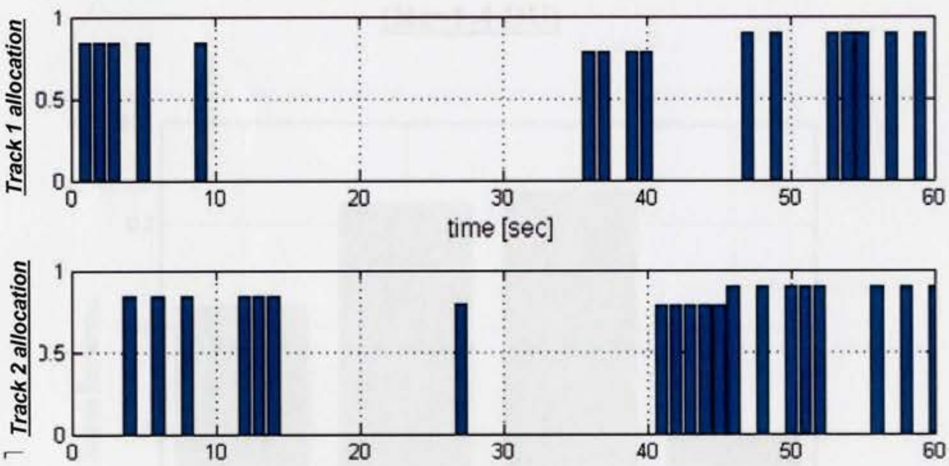

Fig. 7 – Tracking Allocation Under Stressing Conditions (Rc=1.4 DU)

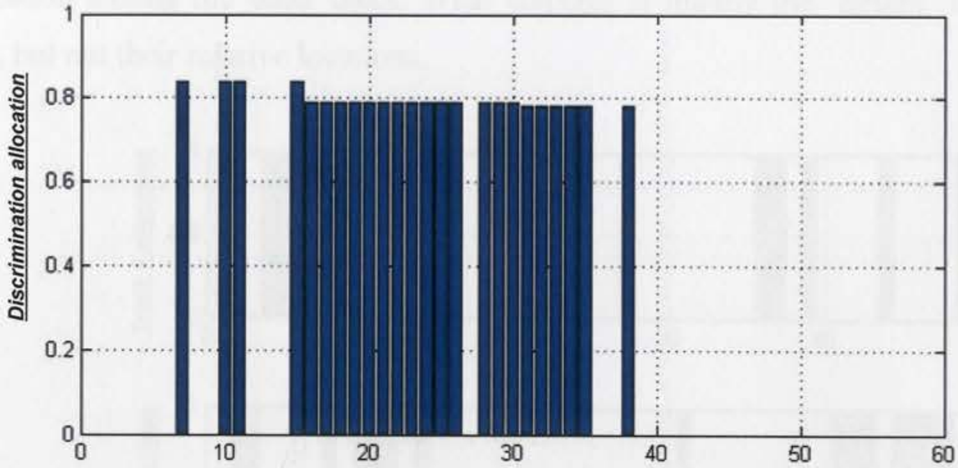

Fig. 8 – Discrimination Allocation Under Stressing Conditions

(Rc=1.4 DU)

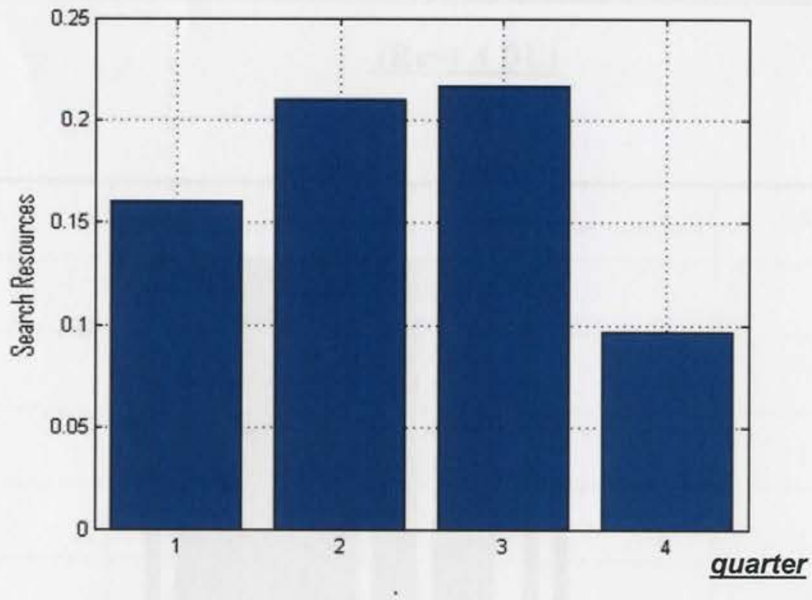

Fig 9 – Search Allocation Under Stressing Conditions (Rc=1.4 DU)

Removing the search mission (assuming our radar is only a tracking radar or equivalently that $P_0 = 1$), we get (Fig. 10 and Fig 11) an almost identical allocation among the other tasks. What changes is mainly the "height" of the bars, but not their relative locations.

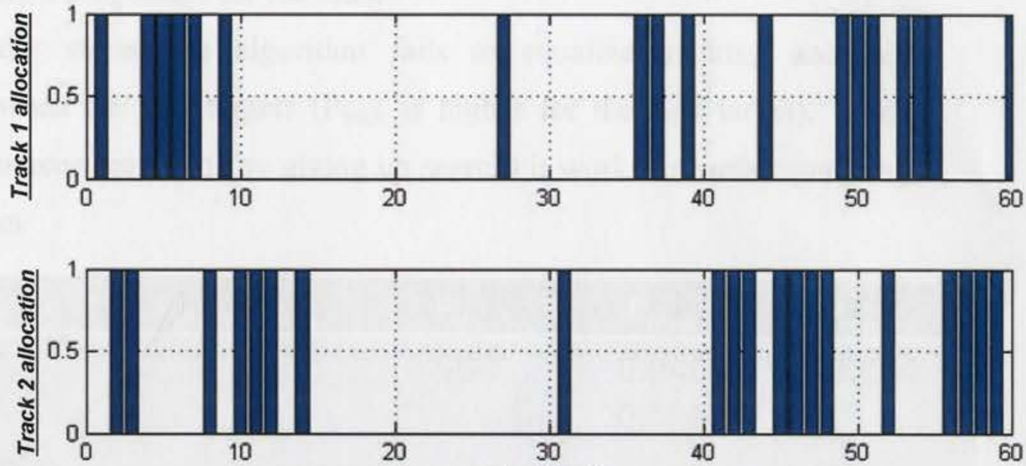

Fig. 10 – Tracking Allocation Under Stressing Conditions (Rc=1.4 DU)

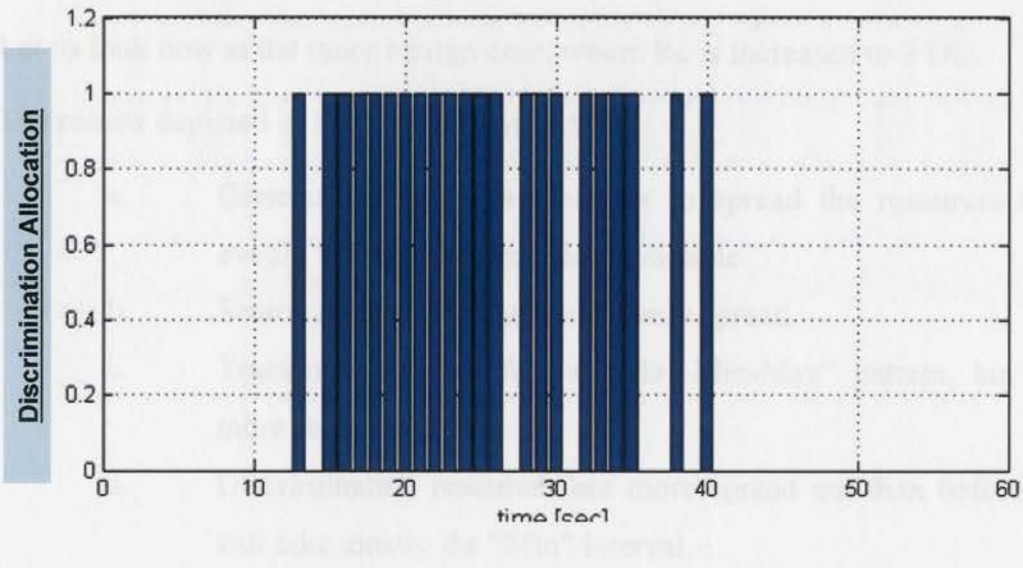

Fig. 11 – Discrimination Allocation Under Stressing Conditions (Rc=1.4 DU)

Table 2 summarizes the results for the stressing scenario. One may observe that:

a. The overall $P_{success}$ in this case is not high, which means that it is really a tough case for the radar.
b. Under stress, the algorithm fails to equalize tracking accuracy between the two targets (P_{track} is higher for the first target). When resources are freed (by giving up search) it works towards equalizing them.

Case	$P_{discrimination}$	P_d	P_{track}	$P_{success}$
p_0=0.3, p_1=0.5, p_2=0.2	0.931	0.980	0.940 0.905	0.764
No search (P_0=1)	0.955	-------	0.945 0.933	0.825

<u>Table 2– Results Summary for the Stressing Scenario</u>

Let us look now at the more benign case, where Rc is increased to 3 DU.

The results depicted in Fig. 12-14 demonstrate:

a. Generally, there is a tendency to spread the resources more evenly through the time slots available.
b. Search resources are almost evenly spread.
c. Tracking resources follow "Max-Min-Max" pattern, but in a more relaxed fashion.
d. Discrimination resources are more spread out than before, but still take mostly the "Min" interval.

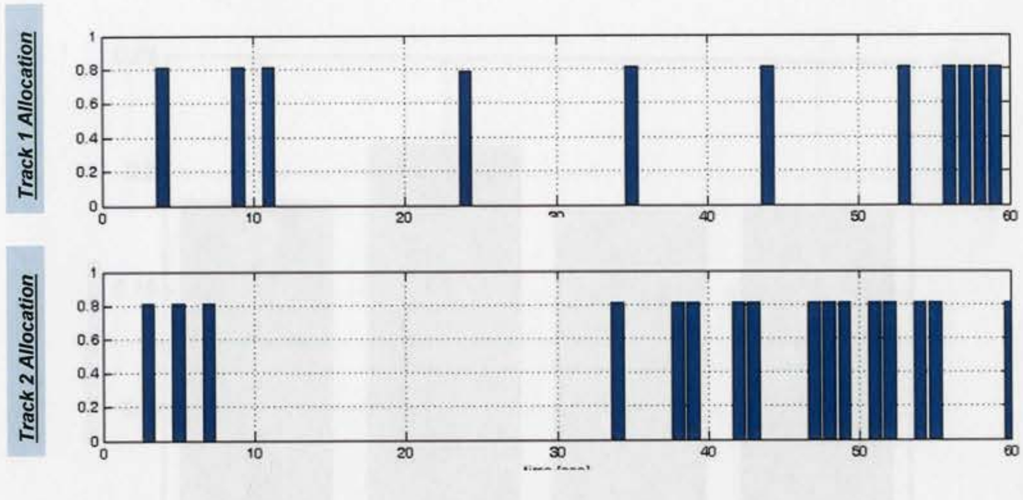

Fig. 12 – Tracking Allocation Under Benign Conditions (Rc=2.1DU)

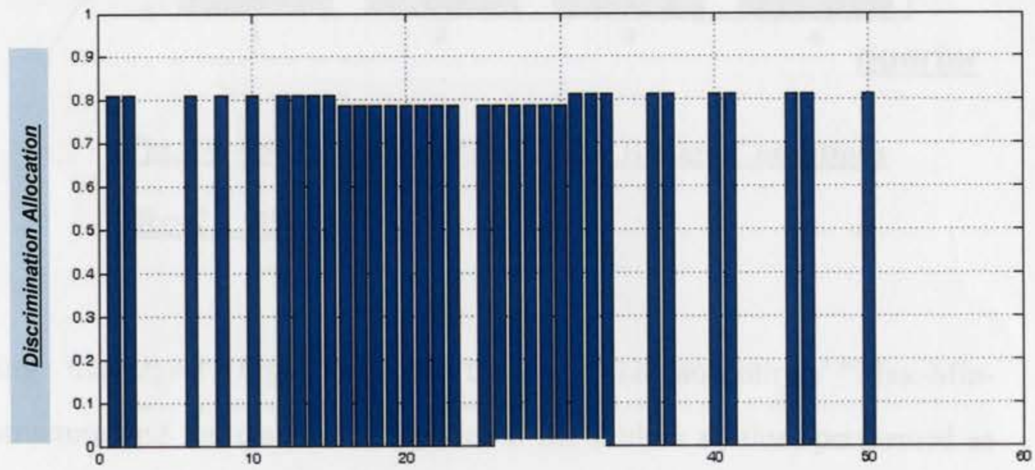

Fig. 13 – Discrimination Allocation Under Benign Conditions (Rc=2.1 DU)

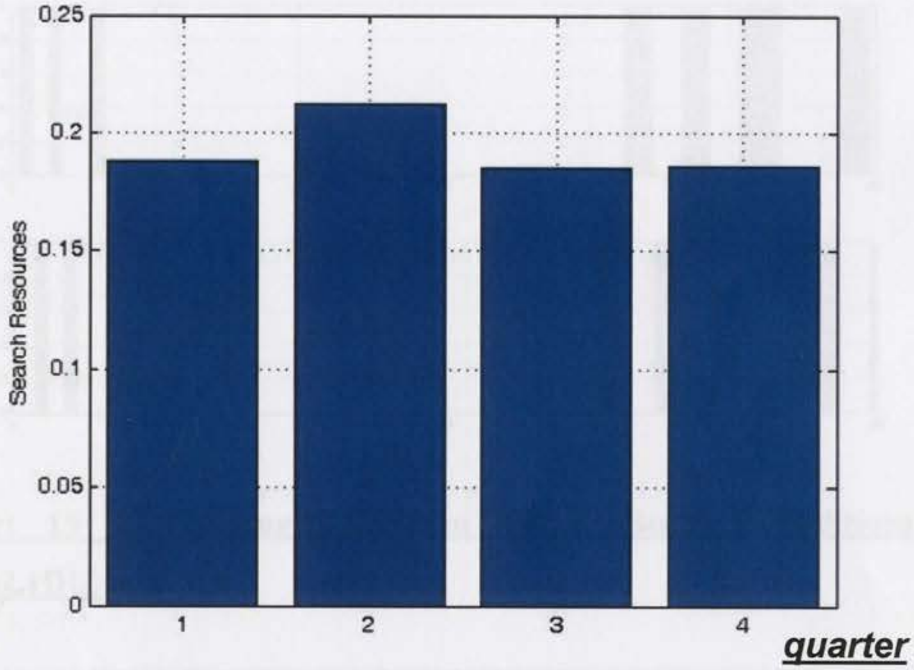

Fig. 14 – Search Allocation Under Benign Conditions (Rc=2.1 DU)

Assuming that $P_0 = 1$ (Fig 15-16), the tracking takes an entirely "Max-Min-Max" structure and the discrimination fits in the middle section, performed as early as possible, following the initial "Max".

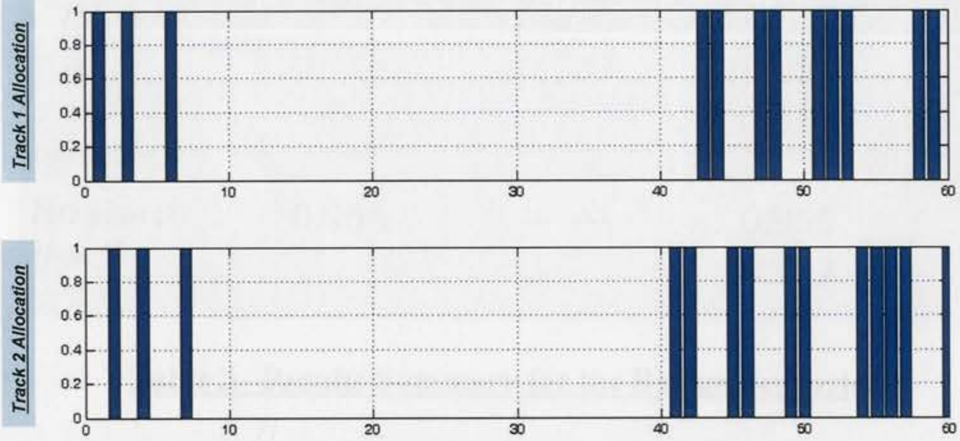

Chart 15 – Tracking Allocation Under Benign Conditions (Rc=2.1DU)

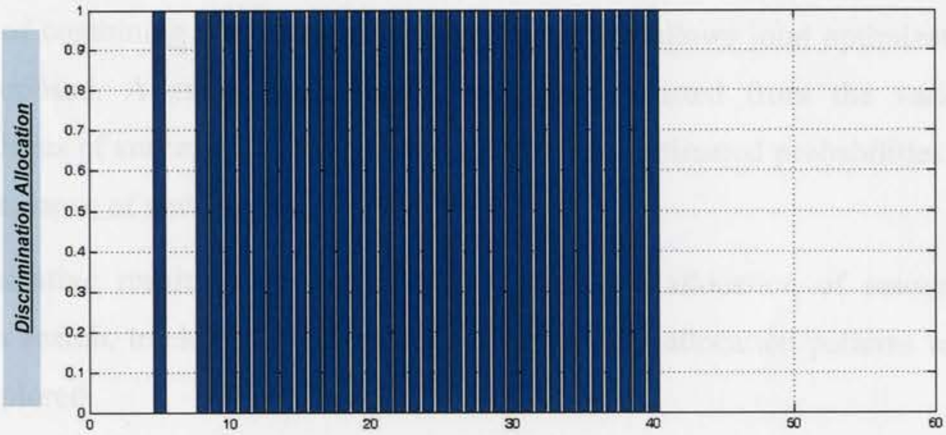

Fig. 16 – Discrimination Allocation Under Benign Conditions (Rc=2.1DU)

Table 3 summarizes the results for the benign case, showing much higher success probabilities, and a return to the familiar trend of equalizing P_{track} between the two tracked targets.

Case	$P_{discrimination}$	P_d	P_{track}	$P_{success}$
p_0=0.3, p_1=0.5, p_2=0.2	0.945	0.987	0.988 0.986	0.905
No search (P_0=1)	0.956	-------	0.995 0.992	0.944

Table 3– Results Summary for the Benign Scenarios

6. Conclusions

A way of combining the different radar missions that allows joint optimization was proposed. A single performance measure is casted from the various probabilities of success. To this end we need to have estimated probabilities for the appearance of new targets.

Representative results were presented regarding the allocation of resources between search, track and discrimination. The optimal allocation patterns have been explored.

The structure of the solution is primarily "Max-Min-Max" for the tracking mission. Time zones of the Min allocation contribute very little to the obtained prediction accuracy and therefore discrimination mission is mostly performed there. Search resources are spread more or less evenly but under stressing conditions they are also shifted to the mid section.

References

1. Ben-Asher J. Z. and Cohen D., “Allocation of Radar Tracking Resources by Direct Optimization”, AIAA J. of Aerospace Computing, Information and Communication, Vol. 6, No. 9, Sept. 2009, pp. 523-539
2. Wintenby J. , “Resource Allocation In Surveillance Radar” , IEEE Report No. 0010684 , 2000, pp. 1-30
3. Kuo T-W, Chao Y-S and Change C , “Real-Time Dwell Scheduling of Component-Oriented Phased Array Radars”, IEEE Transactions on Computers, Vol. 54, No. 1, 2005, pp. 47-60
4. Van-Kuek and Blackman, “On Phased Array Radar Tracking and Parameter Control”, IEEE-Transaction on Aerospace and Electronic Systems, Vol. 29, No. 1, Jan. 1993, pp. 186-190
5. Bar-Shalom et al, “Estimation with Application to Tracking and Navigation”, Wiley, 2001, pp. 57-59
6. Naveh B. and Lorber A., “Theater Ballistic Missile Defense”, Progress in astronautics and Aeronautics Vol. 192 , AIAA, 2001 pp. 118-119
7. Rubinstein Y. R. “The Cross Entropy Method – A unified Approach to combinatorial optimization, Monte-Carlo Simulation and Machine learning,” springer, New-York, 2004, pp. 33-201

Appendix A: Modeling

For the target's dynamics we use a flat earth 2-D scenario governed by the following dynamic equations (Fig. A1):

$$\frac{dx}{dt} = f_1(x,h,v,\gamma) = v \cdot \cos(\gamma)$$
$$\frac{dh}{dt} = f_2(x,h,v,\gamma) = v \cdot \sin(\gamma)$$
$$\frac{dv}{dt} = f_3(x,h,v,\gamma) = -\frac{1}{2}\rho(h)v^2\frac{C_D S}{m} - g \cdot \sin(\gamma) + w_1;$$
$$\frac{d\gamma}{dt} = f_4(x,h,v,\gamma) = -\cos(\gamma) \cdot \frac{g}{v} + \frac{w_2}{v} \qquad \text{(A1)}$$

Where:

x – range

h – altitude

v - velocity

γ – dive angle

m - mass

C_D – drag coefficient

S – reference area

ρ – density

g – gravity

w1,w2 - disturbances

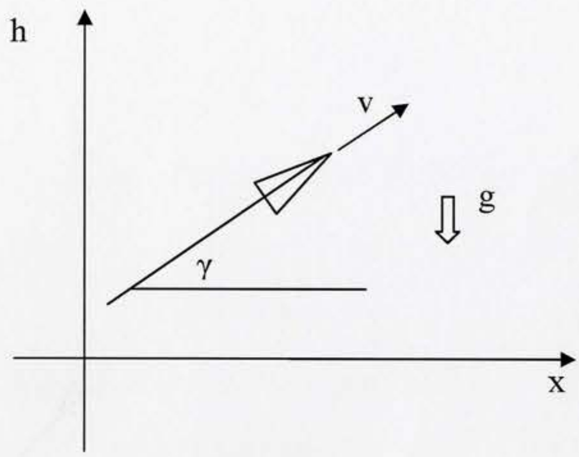

Fig. A1 – States Definition

Let the radar measures slant rage and elevation angle to target.

Thus

$$y_k = \underline{g}_k + \underline{v}_k \quad , \quad \underline{v}_k \sim N(0, R_k)$$

$$\underline{g}_k = \begin{bmatrix} \Theta_k & r_k \end{bmatrix}^T$$

$$\begin{bmatrix} \Theta_k \\ r_k \end{bmatrix} = \begin{pmatrix} \arcsin(\frac{z_{r,k} - z_k}{r_k}) \\ \sqrt{(x_{r,k} - x_k)^2 + (z_{r,k} - z_k)^2} \end{pmatrix} \quad \text{(A2)}$$

Where $( )_k$ denotes values at discrete time k and $( )_{r,k}$ denotes values of radar at this time; x, z are Cartesian coordinates (z=-h); Θ – elevation; r – range.

To obtain the error uncertainty propagation equation, we linearize around a nominal trajectory and we get an expression for the covariance matrix P, thus

$$A(i,k) = \frac{\partial f_i}{\partial x_k} \Delta T + \delta_{ik}; \quad C(i,k) = \frac{\partial g_i}{\partial x_k}; L(i,k) = \frac{\partial f_i}{\partial w_k} \Delta T \quad \text{(A3)}$$

Where δ_{ik} is Kronecker delta and ΔT is the time step of our problem.